Eur. Phys. J. A **26**, 51–59 (2005)
DOI 10.1140/epja/i2005-10151-x

THE EUROPEAN PHYSICAL JOURNAL A

Interplay of direct and spin-flip contributions in Gamow-Teller decay to odd-odd N = Z nuclei

A.F. Lisetskiy[1,a], A. Gelberg[2], and P. von Brentano[2]

[1] National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, MI 48824, USA
[2] Institut für Kernphysik der Universität zu Köln, 50937 Köln, Germany

Received: 3 June 2005 / Revised version: 6 September 2005 /
Published online: 21 October 2005 –
Communicated by R. Krücken

Abstract. Strengths of Gamow-Teller decays of $T_z = \pm 1$ nuclei to $T_z = 0$ odd-odd nuclei have been calculated by using spherical shell model and deformed Nilsson wave functions. The role and competition of the microscopic direct and spin-flip mechanisms generating Gamow-Teller transitions are analyzed. Analytical expressions derived for the B(GT) values give useful insight into the regularities of B(GT) values along the $N = Z$ line. The crucial role of configuration mixing is discussed.

PACS. 23.40.-s β decay; double β decay; electron and muon capture – 21.60.Cs Shell model – 21.60.Ev Collective models

1 Introduction

The Gamow-Teller (GT) beta decay of nuclei with $T = 1$, $J^\pi = 0^+$ to $T = 0, J^\pi = 1^+$ states of odd-odd, $N = Z$ nuclei has been the object of numerous experimental and theoretical investigations (see, for example, [1–8]). The mass number of such nuclei is $A = 4n + 2$, with integer n. The nuclei of this type are characterized by a relatively low-level density [9] and by the existence of $T = 1$ and $T = 0$ excited states in the same energy range [10] , which makes them convenient objects for beta-gamma-ray spectroscopy.

As a matter of fact, most of the strongest known GT decays belong to this category. The classical example is that of ^{6}He, which decays to ^{6}Li with $ft = 813$ ($\log ft = 2.91$) [11]. In a simplified picture, ^{6}He is an α particle plus two neutrons in the same orbital; in fact, it is a halo nucleus. The GT transition is achieved through spin-isospin flip (for total S and T), and the initial and final orbital wave functions have a (nearly) perfect overlap. Such GT transitions are usually called favored allowed or, as in this particular case, even superallowed. This type of decay can be further followed at higher masses. For example, at the beginning of the pf-shell, the ^{42}Ti nucleus decays to ^{42}Sc with $ft = 1562(387)$ [12]. These nuclei have two nucleons outside the doubly magic ^{40}Ca core. The evidence beyond ^{42}Ti is only fragmentary. The experimental investigation of GT decays of heavier nuclei with $T = 1$ can be pursued by using radioactive beams [13].

In the case of some light nuclei, the considerable GT strength of this type of decay can be explained by the Wigner $SU(4)$ symmetry [14,15]. Both the initial and the final state belong to the same irreducible representation (irrep) of $SU(4)$. Since the GT operators $\sigma_i\tau_\pm$ are $SU(4)$ generators, such transitions are allowed. GT transitions between different irreps are forbidden. This situation happens in the decay of $T = 1$ nuclei to even-even, $N = Z$ nuclei ($A = 4n$). In this case, the parent and daughter nuclei do not belong to the the same $SU(4)$ irrep. However, the $SU(4)$ symmetry works only for some selected nuclei in predominantly the lower part of p- and sd-shells. With increasing mass, the $SU(4)$ symmetry starts getting broken by the stronger spin-orbit coupling, and this leads to smaller GT strengths for the decay to $N = Z$, odd-odd nuclei [1].

The properties of odd-odd, $N = Z$ nuclei situated in the pf-shell have been studied in recent years by in-beam gamma-ray spectroscopy [16–23]. The persistence of strong $M1$ transitions for specific configurations in odd-odd $N = Z$ nuclei was one of the important findings [24, 25]. The strong selection rules for the the isovector $\Delta T = 1$ $M1$ transitions in odd-odd $N = Z$ nuclei have been associated with the quasideuteron (QD) degree of freedom, *i.e.*, a proton and a neutron coupled to an even-even $T = 0$ core which is inert with respect to $\Delta T = 1$ excitations. The simple two-nucleon scheme results supported by the full pf-shell model calculations helped to reveal regular features in the structure of the the odd-odd nuclei common for the sd- and pf-shell. Since the spin part of the

[a] e-mail: lisetski@nscl.msu.edu

$M1$ transition operator is proportional to the GT operator the result of previous studies motivated us to look at the GT transitions in the same regions along the $N = Z$ line and to find any similar regularities or consequences of the regularities found for the $M1$ transitions.

The goal of this work is to calculate matrix elements (m.e.) of the GT transition operator from $T = 1$ nuclei to $T = 0$ states of odd-odd, $N = Z$ nuclei in the *framework* of the spherical shell model and deformed Nilsson model, in order to understand the mechanism that governs the behavior of the B(GT) strength for the low-lying states.

2 Two-nucleon configurations

The reduced m.e. for the allowed GT transition between initial $|i\rangle$ state and final $|f\rangle$ state is defined as

$$\langle f\|\sigma\tau\|i\rangle = \langle f\|\sum_k \sigma(k)\tau_{\pm}(k)\|i\rangle, \tag{1}$$

where $\sigma(k)$ and $\tau(k)$ are single-particle spin and isospin operators, respectively, with the convention $\tau_+|n\rangle = |p\rangle$. The sum in eq. (1) is taken over the valence particles.

In the most general case, the GT m.e. (1) can be written down in terms of one-body matrix elements of the GT operator as follows:

$$\langle f\|\sigma\tau\|i\rangle = \sum_{j_1,j_2} \mathcal{D}_{j_1,j_2}(i,f) \tag{2}$$

with

$$\mathcal{D}_{j_1,j_2}(i,f) = n_{1,2}(i,f)\langle j_1\|\sigma\tau\|j_2\rangle, \tag{3}$$

where the sum runs over single-particle orbitals j_i included in the shell model configurational space, $\langle j_1\|\sigma\tau\|j_2\rangle$ is one-particle GT m.e., and $n_{1,2}(i,f)$ are the one-body transition densities defined as

$$n_{1,2}(i,f) = \langle f\|a_1^{\dagger}a_2\|i\rangle. \tag{4}$$

The $n_{1,2}(i,f)$ quantity determines the weight of one-body $\langle j_1\|\sigma\tau\|j_2\rangle$ m.e. in the total m.e. between initial and final states and has the meaning of occupation number of the j-orbital if $i = f$ and $j_1 = j_2 = j$. The one-body GT m.e. $\langle j_1\|\sigma\tau\|j_2\rangle$ is defined as a *direct* one when $j_1 = j_2$:

$$\langle j\|\sigma\tau\|j\rangle = (-1)^{j+l-1/2}\frac{\sqrt{6j(j+1)(2j+1)}}{2l+1}, \tag{5}$$

and as a one-body spin-flip m.e. when $j_1 = l \pm 1/2$ and $j_2 = l \mp 1/2$:

$$\langle j_1\|\sigma\tau\|j_2\rangle = (-1)^{j_2+l-1/2}4\sqrt{\frac{3l(l+1)}{2l+1}}. \tag{6}$$

Let us consider the case of two valence particles and the GT transitions between $0^+, T = 1$ and 1^+, $(T = 0, 1)$ states. If the $J^{\pi} = 0^+$ state is formed by two particles occupying the $j = l + 1/2$ orbital (*i.e.* $|j_>^2; 0^+; T = 1; T_z = 1\rangle$) then there is a direct

$$\langle j_>^2; 1^+_{T=0}\|\sigma\tau\|j_>^2; 0^+_{T=1}\rangle = \sqrt{\frac{4l+6}{2l+1}}, \tag{7}$$

and a spin-flip

$$\langle [j_< \times j_>]; 1^+_{T=0,1}\|\sigma\tau\|j_>^2; 0^+_{T=1}\rangle = \sqrt{\frac{4l}{2l+1}} \tag{8}$$

GT matrix element, where $j_> = l + 1/2$ and $j_< = l - 1/2$. For the case of the initial $|j_<^2; 0^+; T = 1; T_z = 1\rangle$ state, one has

$$\langle j_<^2; 1^+_{T=0}\|\sigma\tau\|j_<^2; 0^+_{T=1}\rangle = -\sqrt{\frac{4l-2}{2l+1}} \tag{9}$$

for the direct m.e., and

$$\langle [j_< \times j_>]; 1^+_{T=0,1}\|\sigma\tau\|j_<^2; 0^+_{T=1}\rangle = \sqrt{\frac{4l+4}{2l+1}} \tag{10}$$

for the spin-flip GT matrix element. It is worth emphasizing that the direct m.e. for the $j_>$ case (eq. (7)) is the largest among all others for the same value of quantum number l.

In both $j_>$ and $j_<$ cases there are two $T = 0$ and one $T = 1$ final $J^{\pi} = 1^+$ states. If there is no configuration mixing, then the values of the GT m.e. are given by eqs. (7)-(10) and the decay to the lowest $T = 0$ state is governed by the direct mechanism entirely, while another $T = 0$ state has a spin-flip character. In the case of the final $T = 1$ state only the spin-flip mechanism is possible. The numerical B(GT) values for the direct transition are shown in the third column of table 1.

In the realistic case of configuration mixing caused by the residual interaction the GT m.e. for the final $T = 0$ state is the superposition of direct and spin-flip terms, while only the spin-flip is possible for the final state with $T = 1$. Therefore the issue of interference between direct and spin-flip contributions concerns mostly the lowest $J^{\pi} = 1^+$ states in daughter odd-odd $N = Z$ nuclei, which are always $T = 0$ states.

Using eqs. (7) and (8) it is easy to check that the summed GT strength is independent of l and amounts to 6 for any residual interaction. We get the same amount in the LS coupling scheme if the $0^+, T = 1$ state is a pure $L = 0$, $S = 0$ state and the $1^+, T = 0$ state has $L = 0$ and $S = 1$. Of course, the GT strength obeys a more general sum rule $\sum B(\mathrm{GT})_- - B(\mathrm{GT})_+ = 3(N - Z)$ [26–28], but in the simple case of two nucleons it yields the same value of 6 [28]. The same is valid for the the initial $\left|j_<^2; T = 1; T_z = 1; J^{\pi} = 0^+\right\rangle$ state.

One may expect that the simple formulas given by eqs. (7)-(10) will work only for the spherical nuclei which can be modelled as an even-even inert core plus two nucleons (or two nucleon holes) occupying the same single j-shell. The results of this extreme approximation are given in the third column of table 1.

Table 1. The B(GT) values for the transitions between $0^+_1, T_z = 1, T = 1$ state of the parent nucleus and $1^+, T_z = 0, T = 0$ state of daughter nucleus. The structure of the final 1^+ states in single-j approximation and corresponding B(GT) values (eqs. (7) and (9)) are shown in the second and third columns, respectively. The shell model results with effective CKI [29], USD [2] and FPD6 [30] interactions for p-, sd- and pf-shells, respectively, are given in fourth and fifth columns (SM). The calculated B(GT) values are given for quenching factor $\alpha_q = 1$ and $\alpha_q = 0.8$. Experimental data and corresponding references are given in the last column.

Daughter	Theory				Expt.	Ref.
nucleus	$(lj)^2$	Eqs. (7), (9)	SM($\alpha_q = 1.0$)	SM($\alpha_q = 0.8$)		
^{6}Li	$p^2_{3/2}$	3.3	5.51	3.53	4.76(1)	[11]
^{14}N	$p^{-2}_{1/2}$	0.7	0.05	0.03	$3.46(3)\cdot10^{-7}$	[31]
^{18}F	$d^2_{5/2}$	2.8	5.06	3.24	3.20(4)	[32]
^{38}K	$d^{-2}_{3/2}$	1.2	0.12	0.08	0.058(8)	[33]
^{42}Sc	$f^2_{7/2}$	2.6	4.84	3.10	2.53(65)	[12]

There is a noticeable difference for the B(GT) values for the $j = l + 1/2$ and $j = l - 1/2$ cases. The shell model results with realistic interactions are given in the fourth column of table 1. If we scale down the shell model values with the appropriate quenching factors we will get satisfactory agreement with experiment (column 5). If we compare the results of the QD scheme (column 3) to the ones of the shell model calculations (column 4) we find that *initially large B(GT) values become larger and small ones almost vanish*. In the shell model the residual interaction naturally mixes direct $j^2_>$, $j^2_<$ and spin-flip $[j_> \times j_<]$ configurations. Shell model calculations show that, for the lower part of any shell, the $j^2_>$ components with maximal l have the largest weight for the $J^\pi = 0^+$ state, while the structure of the lowest $J^\pi = 1^+$ states, in addition to direct $j^2_>$ component, becomes solidly represented by the spin-flip $[j_> \times j_<]$ one. For instance, the shell model wave function with the CKI interaction [29] for the $0^+_1, T = 1$ of ^{6}He has the following form:

$$|0^+, T = 1\rangle = 0.83|p^2_{3/2}; 0^+\rangle + 0.55|p^2_{1/2}; 0^+\rangle, \qquad (11)$$

while for the $1^+_1, T = 0$ state of ^{6}Li one has

$$|1^+, T = 0\rangle = 0.56|p^2_{3/2}; 1^+\rangle + 0.09|p^2_{1/2}; 1^+\rangle + 0.82|[p_{3/2} \times p_{1/2}]; 1^+\rangle. \qquad (12)$$

Calculating the GT m.e. between wave functions (11) and (12) we obtain

$$\begin{aligned}\langle 1^+_1, T = 0\|\sigma\tau\|0^+_1, T = 1\rangle = \\ 0.46\langle p^2_{3/2}; 1^+\|\sigma\tau\|p^2_{3/2}; 0^+\rangle \\ +0.05\langle p^2_{1/2}; 1^+\|\sigma\tau\|p^2_{1/2}; 0^+\rangle \\ +0.68\langle [p_{3/2} \times p_{1/2}]; 1^+\|\sigma\tau\|p^2_{3/2}; 0^+\rangle \\ +0.45\langle [p_{3/2} \times p_{1/2}]; 1^+\|\sigma\tau\|p^2_{1/2}; 0^+\rangle, \qquad (13)\end{aligned}$$

where the first two terms are direct-mechanism contributions and the last two are spin-flip ones. The weight of spin-flip terms is larger than the weight of direct ones; however the spin-flip component is characterized by the GT m.e. (see eq. (8)) that is relatively small (especially for a small l) as compared to the direct one (see eq. (7)). Therefore the individual contributions of each term in eq. (13) are 0.84, −0.03, 0.76 and 0.73, respectively. Thus the contribution of the first direct term (0.84) and its positive interference with the last two spin-flip terms (0.76 and 0.73) plays a very important role in the enhancement of the B(GT) strength. Consequently, *the mechanism of the enhancement is hidden in the interference of the direct and spin-flip contributions*. For the lower part of major shell ($j^2_>$ dominates the 0^+ structure) one has constructive interference of the GT matrix elements (7) and (8) , while for the upper part ($j^2_<$ dominates the structure of the 0^+ state) there is destructive interference of (9) and (10). An appropriate example of the destructive interference is the case of ^{14}N, where the shell model wave functions with the CKI interaction for the $0^+_1, T = 1$ and $1^+_1, T = 0$ states have the following structure:

$$|0^+_{T=1}\rangle = 0.38|p^{-2}_{3/2}; 0^+\rangle + 0.92|p^{-2}_{1/2}; 0^+\rangle, \qquad (14)$$

$$|1^+_{T=0}\rangle = 0.96|p^{-2}_{1/2}; 1^+\rangle + 0.26|[p^{-1}_{3/2} \times p^{-1}_{1/2}]; 1^+\rangle. \qquad (15)$$

The negative sign of the GT m.e. given by eq. (9) leads to the destructive interference of the direct $p^2_{1/2}$ contribution (−0.73) and spin-flip (0.4 and 0.11) ones resulting in a very small B(GT) value of 0.05. However, this small value is still very different from the experimental data (see table 1). This is a well-known problem [34,35] that requires the use of the tensor forces to solve it. Using the example of ^{14}N in the present paper, we aimed only at the schematic illustration of the interference mechanism discussed above.

Systematic regularities originating in an interplay of direct and spin-flip contributions are appropriate for other nuclei shown in table 1 (^{18}F, ^{38}K and ^{42}Sc).

Some other aspects of the problem discussed above may be revealed using the LS coupling scheme instead of the jj one. We turn shortly to the LS coupling below and analyze the same phenomena.

The two-particle wave functions in shell model p-space (the ^{6}Li case) given by eqs. (11), (12) have the following

structure in the LS basis:

$$|0_1^+, T=1\rangle = 0.995|^1S_0\rangle - 0.030|^1P_0\rangle, \quad (16)$$

$$|1_1^+, T=0\rangle = 0.95|^3S_1\rangle + 0.07|^1P_1\rangle + 0.2|^3D_1\rangle, \quad (17)$$

where $^{2S+1}L_J$ labels the state with quantum numbers J, L and S.

Since the $0_1^+, T=1$ and the $1_1^+, T=0$ states are almost completely singlet $^1S_0$ and triplet $^3S_1$ states, respectively, the B(GT) strength is mostly due to the m.e. between these components and may be calculated straightforwardly. This is a two-nucleon spin-flip transition ($S=0$ to $S=1$); however, on the level of one-nucleon degree of freedom this is not a one-nucleon spin-flip (*i.e.* $j_>$ to $j_<$) process, as one may expect, but a strong mixing of direct (*i.e.*, $j_>$ to $j_>$) and spin-flip transitions, as we have shown above.

The structure of the two-nucleon wave functions in the LS representation becomes more complicated when we approach the fp-shell. Thus in the case of shell model calculations for the ^{42}Sc one finds that L and S are not good quantum numbers for the $0_1^+, T=1$ state and the $1_1^+, T=0$ state as well:

$$|0_1^+, T=1\rangle = 0.755|^1S_0\rangle - 0.656|^3P_0\rangle, \quad (18)$$

$$|1_1^+, T=0\rangle = 0.625|^3S_1\rangle + 0.614|^1P_1\rangle + 0.485|^3D_1\rangle. \quad (19)$$

Calculating the GT m.e. between the states (18) and (19) we can identify the contributions of the components with $L=0$ and $L=1$:

$$\langle 1_1^+, T=0\|\sigma\tau\|0_1^+, T=1\rangle = 0.472\langle ^3S_1\|\sigma\tau\|^1S_0\rangle - 0.403\langle ^1P_1\|\sigma\tau\|^3P_0\rangle. \quad (20)$$

If one substitutes the values of partial GT m.e. with $L=0$ and $L=1$ in eq. (20), one notes that they interfere positively:

$$\langle 1_1^+, T=0\|\sigma\tau\|0_1^+, T=1\rangle = 0.711\sqrt{6} + 0.323\sqrt{2}, \quad (21)$$

where the first term corresponds to the $\langle ^3S_3\|\sigma\tau\|^1S_1\rangle$ m.e. and the second to $\langle ^1P_1\|\sigma\tau\|^3P_0\rangle$. Positive interference of two terms in eq. (21) results in a large B(GT) value of 4.84.

This type of constructive interference is closely related to the positive interference of the direct $f_{7/2}^2$ and spin-flip $[f_{7/2}\times f_{5/2}]$ GT contributions and has the same origin as the interference of the orbital and spin parts of the $M1$ m.e. [24]. This stems from the fact that the spin and orbital angular momentum for the $f_{7/2}$ orbital ($j_>$ type) are aligned.

In the case of two holes in p-shell (^{14}N) or sd-shell (^{38}K) the shell model wave functions of the 0^+ state is dominated by the $j_<$ orbital components (see *e.g.*, eqs. (14), (15)). This translates, for example for ^{38}K, to the following structure in LS representation:

$$|0_1^+, T=1\rangle = 0.65|^1S_0\rangle + 0.76|^3P_0\rangle, \quad (22)$$

$$|1_1^+, T=0\rangle = 0.46|^3S_1\rangle + 0.29|^1P_1\rangle + 0.84|^3D_1\rangle. \quad (23)$$

The situation in the upper part of the sd-shell (^{38}K) is strikingly similar to the ^{14}N case [34]. The lowest $J=1^+, T=0$ state has a large triplet $^3D_1$ component (70%) which does not contribute to the GT transition. Furthermore, there is a destructive interference of the singlet $^1P_1$ (9%) and triplet $^3S_1$ (21%) contributions that results in a tiny B(GT) value. The destructive interference of $^1P_1$ and $^3S_1$ contributions at the upper part of the sd-shell corresponds to the negative interference of direct $d_{3/2}^2$ and spin-flip $[d_{3/2}\times d_{5/2}]$ contributions in terms of one-body m.e. as we have shown above. This phenomenon is related to the fact that the spin and orbital momentum of the $j_<$ orbital are anti-aligned. This leads also to the cancellation of orbital and spin parts of $M1$ operator for the isovector transitions in odd-odd $N=Z$ nuclei [24].

In the following section we would like to explore an interplay of direct and spin-flip mechanism for the case of many-nucleon configurations.

3 Deformed configurations

One has a more complicated situation in the case of $n_{\rm val}=4n+2$ valence particles with $n\geq 1$ along the $N=Z$ line. The accumulated proton-neutron interaction in the considered case rapidly (especially in the case of light nuclei) leads to the increase of quadrupole deformation. This fact allows one to use the collective rotational model for the analysis of GT-transitions in deformed nuclei along the $N=Z$ line. One of the useful and simple models in which one can address this question is the particle-plus-rotor model (PRM) [36]. This model gives nice interpretation of the GT transitions for deformed odd-A nuclei (see [36], p. 306) and one may expect the same for the odd-odd nuclei.

In the next subsection we present the main idea of the model and show how the GT matrix elements calculated for deformed states may be decomposed into contributions of one-body GT m.e. for spherical orbitals.

3.1 Particle-plus-rotor model

The basic assumption of the model [37,38] is that one has two nucleons outside an even-even deformed rotating core. We consider the simplified version of this model neglecting the Coriolis interaction and the residual interaction between nucleons in the two-nucleon cluster, however explicitly treating the isospin degree of freedom. Then the states of the nucleus are characterized by the quantum numbers $JMKT$ and the total wave function has the form appropriate to a rotationally invariant system with axial symmetry which also possesses the signature symmetry [36]:

$$|JMKT\rangle = \sqrt{\frac{2J+1}{16\pi^2(1+\delta_{K,0})}} \times \left[D^J_{MK}\Phi_{K,T} + (-1)^{J+K}D^J_{M-K}\Phi_{\overline{K},T}\right]. \quad (24)$$

The wave function in the intrinsic system is

$$\Phi_{K,T} = \left[\frac{u_a(1)u_b(2) + (-)^T u_a(2)u_b(1)}{2}\right]\zeta^T_{T_z}(1,2), \quad (25)$$

where $u_{\rho\equiv\{\Omega,\Lambda,n_z,t_z\}}(i)$ are single-particle eigenfunctions of the Nilsson Hamiltonian, Ω_ρ is the 3-projection of particle angular momentum, $K = \Omega_a + \Omega_b$ and $\zeta^T_{T_z}(1,2)$ is the isospin wave function with the isospin quantum numbers T and $T_z = t_z(1) + t_z(2)$. The states belonging to a $K = 0^+$ band have only even (odd) spins for the signature quantum number $r = +1$ (-1). It can be shown that states belonging to the $T = 1$ $(T = 0)$ band have $r = +1$ (-1). The coupling of the angular momenta of the two-nucleon subsystem and the rotor, that is implicit in eq. (24), can be exhibited by transforming to the representation of explicit coupling of angular momenta [36] appropriate for a strongly coupled system. This representation, which can be treated also as an algebraic representation [39], allows one to work with spherical shell model configurations.

As a starting point we use particle-plus-rotor model basis states written in terms of spherical single-particle wave functions in a strong coupling approximation [36,39]:

$$|JMK\rangle = \sqrt{1+\delta_{K_R,0}} \times \sum_{R,j}\sqrt{\frac{(2R+1)}{2J+1}}C^{JK}_{RK_Rj\Omega}\chi^\Omega_j\left[|R\rangle\otimes|j\rangle\right]^J_M, \quad (26)$$

where $C^{JK}_{RK_Rj\Omega}$ is the Clebsch-Gordan coefficient, χ^Ω_j are projection coefficients of single-particle Nilsson orbitals with quantum numbers $[Nn_z\Lambda]\Omega$ on the spherical single particle $|nlj\Omega\rangle$ basis [40]:

$$|Nn_z\Lambda;\Omega\rangle = \sum_{j=\Omega}^{N+1/2}\chi^\Omega_j|nlj\Omega\rangle, \quad (27)$$

R is the core angular momentum quantum number and

$$\left[|R\rangle\otimes|j\rangle\right]^J_M = \sum_{M_R,m}C^{JM}_{RM_Rjm}|RM_R\rangle\cdot|nljm\rangle$$

are weakly coupled particle-plus-rotor states. The wave functions for two-particle states coupled to the $K_R = 0, T = 0$ rotational core can be easily constructed by applying eq. (26). After some simple transformations one obtains

$$|JMKTT_z\rangle = \sum_{R,J_q}\sqrt{\frac{2(2R+1)}{2J+1}}C^{JK}_{R0J_qK}\left[|R\rangle\otimes|J_q\rangle\right]^{JT}_{MT_z}, \quad (28)$$

where $|J_q\rangle$ is a two-nucleon state in the deformed field:

$$|J_q\rangle \equiv |J_qM_qKTT_z\rangle = \sum_{j_1,j_2}\chi^{\Omega_1}_{j_1}\chi^{\Omega_2}_{j_2}C^{J_qK}_{j_1\Omega_1j_2\Omega_2}\left[|j_1\rangle\otimes|j_2\rangle\right]^{J_qT}_{M_qT_z}, \quad (29)$$

and Ω_i is the Nilsson quantum number of angular momentum projection on the symmetry axis for the nucleons building the two-nucleon cluster. Let us suppose that the intrinsic structure of the initial $0^+, K^\pi = 0^+$ state is determined by the following Nilsson orbitals: $\left|Nn_z^{(1)}\Lambda_1;\Omega_1\right\rangle \times \left|Nn_z^{(2)}\Lambda_2;\Omega_2\right\rangle$ $(\Omega_1 = -\Omega_2)$ while the final $1^+, T, K = \Omega_3 \pm \Omega_4$ states are characterized by the following structures: $\left|Nn_z^{(3)}\Lambda_3;\Omega_3\right\rangle \times \left|Nn_z^{(4)}\Lambda_4;\Omega_4\right\rangle$. We then obtain the following formula for the B(GT) values for the transitions between these initial and final states:

$$B(GT;0^+_{K=0}\to 1^+_K) = \frac{4}{9}\left[\sum_{j_1,j_3}\chi^{\Omega_1}_{j_1}\chi^{\Omega_3}_{j_3}C^{1K}_{j_1\Omega_1j_3\Omega_3}\frac{\langle j_1\|\sigma\tau\|j_3\rangle}{\sqrt{2-\delta_{1,3}}}\right]^2. \quad (30)$$

The terms with $j_1 = j_3$ in eq. (30) represent the direct mechanism while the terms with $j_1 \neq j_3$ correspond to the one-body spin-flip mechanism. The derived eq. (30) is analogous to eq. (2), where the one-body transition densities $n_{1,3}(0^+_1, 1^+_1)$ have the following simple form:

$$n_{1,3}(0^+_1,1^+_1) = \frac{\chi^{\Omega_1}_{j_1}\chi^{\Omega_3}_{j_3}C^{1K}_{j_1\Omega_1j_3\Omega_3}}{\sqrt{2-\delta_{1,3}}}. \quad (31)$$

3.2 Results and comparison with shell model

It is instructive to analyze eq. (30) taking into account a dominant spherical component in the Nilsson state, *i.e.*, to put $\chi^\Omega_j = 1$ for the dominant $j \equiv j^d$-component. Following this prescription we get the set of simple formulas

$$B(GT;0^+\to 1^+_{T=0},K) = \frac{4l+6}{2l+1}\left[C^{j^d_1\Omega_1}_{j^d_3\Omega_31K}\right]^2 \quad \text{for } j^d_1 = j^d_3 = l+1/2, \quad (32)$$

$$\frac{4l-2}{2l+1}\left[C^{j^d_1\Omega_1}_{j^d_3\Omega_31K}\right]^2 \quad \text{for } j^d_1 = j^d_3 = l-1/2, \quad (33)$$

Equations (32) and (33) correspond to the direct transition mechanism for deformed states and differ from eqs. (7) and (9) for the spherical case only by the Clebsch-Gordan coefficients. The spin-flip GT transitions are described by

$$B(GT;0^+\to 1^+_{T=0,1},K) = \frac{4l}{2l+1}\left[C^{j^d_1\Omega_1}_{j^d_3\Omega_31K}\right]^2 \quad \text{for } j^d_1 = j^d_3+1 = l+1/2, \quad (34)$$

$$\frac{4(l+1)}{2l+1}\left[C^{j^d_1\Omega_1}_{j^d_3\Omega_31K}\right]^2 \quad \text{for } j^d_1 = j^d_3-1 = l-1/2. \quad (35)$$

Similarly to the direct mechanism, the spin-flip B(GT) values (34) and (35) for the deformed states differ from the corresponding ones for two-nucleon spherical states by the Clebsch-Gordan coefficient too.

Table 2. B(GT) values for the transitions between $0^+_1, T_z = 1, T = 1, K = 0$ state and first $1^+_1, T_z = 0, T = 0, K = 0$ states. The intrinsic structure of the final 1^+_1 states is shown in the third column, where $[lj]$ indicates the dominant spherical component in eq. (27). Calculations are performed for effective quadrupole deformation $\beta_{\rm eff}$ (column 2). Effective USD [2] interaction was used in the shell model calculations for the *sd*-shell nuclei. Effective KB3G [41] and KB3 [19] interactions were used for ^{46}V and ^{50}Mn, respectively. The B(GT) values are shown for the quenching parameters $\alpha_q = 1.0$ and $\alpha_q = 0.77$. Experimental data and corresponding references are given in the last column.

Daughter nucleus	$\beta_{\rm eff}$	Structure $([lj], \Omega)^2$	B(GT;$0^+ \to 1^+$)						
			Theory					Expt.	Ref.
			Eqs. (32), (33)	PRM, Eq. (36)		Shell model (SM)			
			$\alpha_q = 1.0$	$\alpha_q = 1.0$	$\alpha_q = 0.77$	$\alpha_q = 1.0$	$\alpha_q = 0.77$		
^{22}Na	0.43	$([d_{5/2}], 3/2)^2$	0.72	1.38	0.82	1.89	1.12	0.85(2)	[2]
^{26}Al	0.38	$([d_{5/2}], 5/2)^2$	2.00	2.00	1.18	1.91	1.13	1.08(2)	[33]
^{34}Cl	0.23	$([d_{3/2}], 3/2)^2$	0.72	0.40	0.24	0.009	0.005	0.0188(22)	[2]
^{46}V	0.23	$([f_{7/2}], 3/2)^2$	0.37	0.71	0.42	0.549	0.325	0.64(17)	[42]
^{50}Mn	0.25	$([f_{7/2}], 5/2)^2$	1.02	1.34	0.79	1.31	0.78	0.59(16)	[43]
^{54}Co	0.16	$([f_{7/2}], 7/2)^2$	2.00	2.00	1.18			0.67(15)	[44]
^{66}As	0.23	$([f_{5/2}], 3/2)^2$	0.37	0.36	0.22				
^{70}Br	0.25	$([p_{3/2}], 3/2)^2$	2.00	1.49	0.88				
^{74}Rb	0.25	$([f_{5/2}], 5/2)^2$	1.02	0.81	0.48				

The above-derived formulas give a qualitative insight into the general tendency for the distribution of Gamow-Teller strength based only on the information about quantum numbers of Nilsson orbitals closest to the Fermi surface. These formulas indicate that the B(GT) values are proportional to Ω^2 for the lowest quasideuteron states $J^\pi = 1^+, T = 0, K = 0$, which are characterized by identical proton and neutron quantum numbers. Subsequently, the largest portion of the strength is expected for the states characterized by the highest value of Ω, where it is completely due to the direct mechanism.

The lowest $J = 1^+_1, T = 0$ state in odd-odd daughter nucleus is of special interest. If $K = 0$ and $\Omega \neq 1/2$ (*i.e.* when there is no mixing between $K = 0$ and $K = 1$ states caused by the Coriolis interaction) for the $J = 1^+_1, T = 0$ state we obtain

$$B(GT; 0^+ \to 1^+) = 2 \left(\sum_{l=l_{\rm min}}^{N} \frac{\Omega}{j_>} G(\Omega, l) \right)^2, \qquad (36)$$

where $l_{\rm min} = \Omega + \frac{1}{2}(-)^N$, and

$$G(\Omega, l) = \left(\chi^\Omega_{j_>}\right)^2 + \left(\chi^\Omega_{j_<}\right)^2 - \sqrt{2}\chi^\Omega_{j_<}\chi^\Omega_{j_>}\sqrt{\left(\frac{j_>}{\Omega}\right)^2 - 1}. \qquad (37)$$

The character of the interference (positive or negative) between direct and spin-flip contributions in eq. (37) depends on the amplitudes of spin-orbit partners from the single-particle Nilsson wave function. There may be also another type of the interference between $G(\Omega, l)$ terms with different orbital quantum number l. The B(GT) values calculated using eq. (36) are listed in column PRM in table 2. We have also performed shell model calculations in full *sd*- and *pf*-spaces for some of the nuclei. The B(GT) values for different quenching are listed in column SM in table 2.

Comparing the single-j deformed orbital approximation results (column 4) and exact Nilsson wave function results (column 5) without quenching ($\alpha_q = 1.0$) one notices that B(GT) values are large in the lower part of the *sd*- or *pf*-shell while for the upper part one observe a considerable decrease of the GT strength. It is noticeable that the configuration mixing caused by the enhanced quadrupole correlations acts constructively for the lower part and destructively for the upper part as occurs for the two-nucleon spherical configurations discussed in the previous section.

If we take, for example, ^{22}Na and ^{34}Cl nuclei (*sd*-shell), or ^{46}V and ^{66}As nuclei (*pf*-shell), we have the same B(GT) for each pair of nuclei in the single-j deformed orbital approach (column 5). However, the configuration mixing increases the strength for the ^{22}Na and ^{46}V, while a decrease is observed for the ^{34}Cl and the ^{66}As. This mechanism may be easily seen from eq. (37). The first and second terms have to be associated with the direct mechanism while the third one is of spin-flip character. The third term in this formula depends on the relative sign of the amplitudes of single-particle spin-flip component $\chi^\Omega_{j_<}\chi^\Omega_{j_>}$. For the ^{22}Na the $\Omega = 3/2_1$ Nilsson state is a superposition of $d_{5/2}$ and $d_{3/2}$ orbitals with amplitudes that have opposite sign. For the ^{34}Cl case one has a Nilsson $\Omega = 3/2_2$ state which is orthogonal to the $\Omega = 3/2_1$ state (assuming the same deformation for both of them) and thus the amplitudes of $d_{5/2}$ and $d_{3/2}$ orbitals have the same sign. Similar considerations are valid for the *pf*-shell examples mentioned above.

Comparing the results of the collective model with those of the shell model we note that the B(GT) values are rather similar and the regularities appropriate for

Table 3. Partial contributions $\mathcal{D}_{j_1,j_2}(0_1^+,1_1^+)$ (eq. (3)) of direct and spin-flip mechanisms to the GT matrix element between $0_1^+, T_z = 1, T = 1$ state and $1_1^+, T_z = 0, T = 0$ states for *sd*-shell nuclei. Results are shown for shell model (SM) (eq. (2)) with the USD interaction [2] and collective particle-plus-rotor model (PRM) (eq. (36)) with corresponding value of quadrupole deformation given in table 2. The total GT m.e. is shown in the last column.

Daughter nucleus	Model	$\mathcal{D}_{j_1,j_2}(0_1^+,1_1^+)$				$M_{\rm GT}(0^+\to 1^+)$
		Direct, $j\to j$			Spin-flip, $j_>\to j_<$	
		$d_{5/2}d_{5/2}$	$d_{3/2}d_{3/2}$	$s_{1/2}s_{1/2}$	$d_{3/2}d_{5/2}$	
^{22}Na	SM	0.847	−0.032	−0.073	0.634	1.376
	PAM	0.812	0.037	0.0	0.326	1.175
^{26}Al	SM	1.285	0.045	0.244	−0.196	1.382
	PAM	1.414	0.0	0.0	0.0	1.414
^{34}Cl	SM	0.016	0.473	−0.224	−0.357	−0.092
	PAM	0.016	0.830	0.0	−0.215	0.634

Table 4. Partial contributions $\mathcal{D}_{j_1,j_2}(0_1^+,1_1^+)$ (eq. (3)) of direct and spin-flip mechanisms to the GT matrix element between $0_1^+, T_z = 1, T = 1$ state and $1_1^+, T_z = 0, T = 0$ states for pf-shell nuclei. Results are shown for shell model (SM) (eq. (2)) with the KB3G interaction [41] for ^{46}V. Collective PRM results (eq. (36)) with corresponding value of quadrupole deformation given in table 2. The total GT m.e. is shown in the last column.

Daughter nucleus	Model	$\mathcal{D}_{j_1,j_2}(0_1^+,1_1^+)$						$M_{\rm GT}(0^+\to 1^+)$
		Direct, $j\to j$				Spin-flip, $j_>\to j_<$		
		$f_{7/2}f_{7/2}$	$f_{5/2}f_{5/2}$	$p_{3/2}p_{3/2}$	$p_{1/2}p_{1/2}$	$f_{7/2}f_{5/2}$	$p_{3/2}p_{1/2}$	
^{46}V	SM	0.710	−0.011	−0.002	0.011	0.141	−0.108	0.741
	PAM	0.568	0.008	0.070	0.0	0.198	0.0	0.843
^{50}Mn	PAM	0.999	0.012	0.0	0.0	0.149	0.0	1.159
^{54}Co	PAM	1.414	0.0	0.0	0.0	0.0	0.0	1.414

collective model are qualitatively reproduced by the shell model and by the experimental data.

The puzzling similarity of collective and shell model results motivates us to look at different direct and spin-flip contributions to the total GT strength for both models. Partial one-body contributions to the GT m.e. are shown in tables 3 and 4. Comparing the results of the two models, one may see that the partial contributions of different mechanisms are similar for the two approaches as well. Since only the dynamics of the outer two nucleons determine the character and strength of GT transitions in collective particle-plus-rotor model one may come to the conclusion that the same is appropriate for exact microscopic shell model calculations that take into account all possible degrees of freedom of many-body systems of valence nucleons. As noticed in sect. 2, in the case when the number of valence nucleons $n_v > 2$, an approximation considering only two nucleons violates the Pauli principle. However, the successful applications of this scheme, *e.g.* to the PRM, shows that the violation of the Pauli principle must be weak. In particular, the fact that protons and neutrons populate successive Nilsson orbitals with different values of Ω takes into account, to a certain extent, the Pauli principle. It is interesting that sophisticated effective shell model interactions yield the same results as the schematic Nilsson model with only quadrupole interaction. To get deeper understanding of the problem it has to be elaborated farther.

4 Conclusion

In conclusion, the role of the direct and spin-flip mechanisms for the GT transitions from the $J^\pi = 0^+$, $T = 1$, $T_z = 1$ state to the $J^\pi = 1^+$, $T = 0$, $T_z = 0$ states of the odd-odd $N = Z$ nucleus are analyzed in this work. We have derived simple analytical expressions for the GT transition strength that allow straightforward calculations illustrating the role of the different two-nucleon configurations. The comparison of the single-j and exact shell model diagonalization results in the case of spherical two-nucleon configurations helps to understand the mechanism leading to the enhancement or quenching of the GT transition strength for different nuclei. Our studies show that neither direct, $j_> \to j_>$ and $j_< \to j_<$, nor spin-flip, $j_> \to j_<$, single-particle processes, but rather their specific combination determines the lowest GT strength. The mixing of the configurations induced by the residual interaction plays a decisive role for the exact B(GT) value. However, the j^2 content of the initial $J^\pi = 0_1^+$, $T = 1, T_z = 1$ state itself gives a simple rule for the lowest GT strength when $T = 0$ residual interaction for the $J^\pi = 1_1^+$, $T = 0, T_z = 0$ state is switched on. If the $j_>^2$ component is dominant for the initial $J^\pi = 0_1^+, T = 1$ state, which usually occurs for the lower part of any major shell, then the residual interaction induces strong positive interference between the direct $j_>^2$ and spin-flip $j_> \times j_<$ contributions that results in enhanced B(GT) values. But if the $j_<^2$ component of the

initial $J^\pi = 0^+_1, T = 1$ state has larger weight (this usually occurs for the upper part of major shell) then the residual interaction induces destructive interference between the direct $j^2_<$ and spin-flip $j_> \times j_<$ contributions, yielding vanishing B(GT) values.

In the case of the lower part of p- and sd-shell this mechanism leads to a situation similar to the $SU(4)$ symmetry where the strong B(GT) is caused by the dominant $L = 0, S = 0$ structure of the 0^+ state and the $L = 0, S = 1$ structure of the 1^+ state. In the case of stronger spin-orbital interaction when the $SU(4)$ symmetry is broken, *i.e.* for upper part of sd-shell and all fp-shell, the interference of direct and spin-flip contributions translates into the interference of the $\langle {}^1S_1 \| \sigma\tau \| {}^3S_3 \rangle$ and $\langle {}^3P_1 \| \sigma\tau \| {}^1P_3 \rangle$ m.e.'s producing a similar strong effect.

We have extended the two-nucleon scheme to the deformed nuclei to verify the identified mechanism. The Nilsson single-particle states which are represented by a mixture of various spherical single-particle states due to the assumed quadrupole interaction, yield similar results. We observe the same interference between direct and spin-flip contributions that changes from positive for the lower part of the shell to negative at the upper part. The results of simple scheme is well supported by large-scale shell model calculations with various effective interactions. We consider it as an indication of the crucial role of the outer two-nucleon degree of freedom in the β decay to the lowest $J^\pi = 1^+, T = 0$ state of odd-odd $N = Z$ nuclei.

The understanding of the generating mechanism of GT transitions for sd-shell and lower-part pf-shell nuclei may be helpful for a qualitative analysis of the situation along the $N = Z$ line at the upper part of the pf-shell [45], where one has to deal with proton-rich nuclei and related astrophysical fp-process.

The authors want to thank A. Arima, A. Brown, R.F. Casted, A. Dade, F. Iachello, J. Jolie, T. Otsuka, N. Summers, I. Talmi, L. Zamick and V. Zelevinsky for stimulating discussions and suggestions. This work was supported in part by the DFG under support No. Br -799 /12-1 and the U.S. National Science Foundation Grant No. PHY-0244453.

References

1. P. Van Isacker, Rep. Progr. Phys. **69**, 1661 (1999).
2. B.A. Brown, B.H. Wildenthal, At. Data Nucl. Data Tables **33**, 347 (1985).
3. A. Arima, K. Shimizu, W. Bentz, H. Hyuga, Adv. Nucl. Phys. **18**, 1 (1987).
4. B.A. Brown, B.H. Wildenthal, Annu. Rev. Nucl. Part. Sci. **38**, 29 (1988).
5. Y. Fujita, H. Fujita, T. Adachi, G.P.A. Berg, E. Caurier, H. Fujimura, K. Hara, K. Hatanaka, Z. Janas, J. Kamiya, T. Kawabata, K. Langanke, G. Martnez-Pinedo, T. Noro, E. Roeckl, Y. Shimbara, T. Shinada, S.Y. van der Werf, M. Yoshifuku, M. Yosoi, R.G.T. Zegers, Eur. Phys. J. A **13**, 411 (2002).
6. Y. Fujita, Y. Shimbara, A.F. Lisetskiy, T. Adachi, G.P.A. Berg, P. von Brentano, H. Fujimura, H. Fujita, K. Hatanaka, J. Kamiya, T. Kawabata, H. Nakada, K. Nakanishi, Y. Shimizu, M. Uchida, M. Yosoi, Phys. Rev. C **67**, 064312 (2003) and references therein.
7. K. Langanke, D.J. Dean, P.B. Radha, Y. Alhassid, S.E. Koonin, Phys. Rev. C **52**, 718 (1995).
8. T. Suzuki, R. Fujimoto, T. Otsuka, Phys. Rev. C **67**, 044302 (2003).
9. R.R. Chasman, Phys. Lett. B, **553**, 204 (2003).
10. P. von Brentano, A.F. Lisetskiy, I. Schneider, C. Friessner, R.V. Jolos, N. Pietralla, A. Schmidt, Prog. Part. Nucl. Phys. **44** 29 (2000).
11. K. Riisager, M.J.G. Borge, H. Gabelmann, P.G. Hansen, L. Johannsen, B. Jonson, W. Kurcewicz, G. Nyman, A. Richter, O. Tengblad, K. Wilhelmsen, and the ISOLDE Collaboration, Phys. Lett. B, **235**, 31 (1990).
12. Balraj Singh, John A. Cameron, Nucl. Data Sheets **92**, 1 (2001).
13. E. Roeckl, Eur. Phys. J. A **15**, 139 (2002).
14. E.P. Wigner, Phys. Rev. **51**, 96 (1937).
15. I. Talmi, *Simple Models of Complex Nuclei* (Harwood Academic, Chur, 1993).
16. C. Frießner, N. Pietralla, A. Schmidt, I. Schneider, Y. Utsuno, T. Otsuka, P. von Brentano, Phys. Rev. C **60**, 011304 (1999).
17. S.M. Lenzi, D.R. Napoli, C.A. Ur, D. Bazzacco, F. Brandolini, J.A. Cameron, E. Caurier, G. de Angelis, M. De Poli, E. Farnea, A. Gadea, S. Hankonen, S. Lunardi, G. Martinez-Pinedo, Zs. Podolyak, A. Poves, C. Rossi Alvarez, J. Sanchez-Solano, H. Somacal, Phys. Rev. C **60**, 021303 (1999).
18. I. Schneider, A.F. Lisetskiy, C. Frießner, R.V. Jolos, N. Pietralla, A. Schmidt, D. Weißhaar, P. von Brentano, Phys. Rev. C **61**, 044312 (2000).
19. N. Pietralla, R. Krücken, C.J. Barton, C.W. Beausang, M.A. Caprio, R.F. Casten, J.R. Cooper, A.A. Hecht, J.R. Novak, N.V. Zamfir, A. Lisetskiy, A. Schmidt, Phys. Rev. C **65**, 024317 (2002).
20. O. Möller, K. Jessen, A. Dewald, A.F. Lisetskiy, P. von Brentano, A. Fitzler, J. Jolie, A. Linnemann, B. Saha, K.O. Zell, Phys. Rev. C **67**, 011301(R) (2003).
21. D.G. Jenkins, N.S. Kelsall, C.J. Lister, D.P. Balamuth, M.P. Carpenter, T.A. Sienko, S.M. Fischer, R.M. Clark, P. Fallon, A. Görgen, A.O. Macchiavelli, C.E. Svensson, R. Wadsworth, W. Reviol, D.G. Sarantites, G.C. Ball, J. Rikovska-Stone, O. Juillet, P. Van Isacker, A.V. Afanasjev, S. Frauendorf, Phys. Rev. C **65**, 064307. (2002).
22. A.F. Lisetskiy, N. Pietralla, M. Honma, A. Schmidt, I. Schneider, A. Gade, P. von Brentano, T. Otsuka, T. Mizusaki, B.A. Brown, Phys. Rev. C **68**, 034316 (2003).
23. D. Rudolph, C. Andreoiu, J. Ekman, C. Fahlander, M.N. Mineva, S.M. Lenzi, E. Farnea, C. Rossi-Alvarez, C.A. Ur, M. Axiotis, G. de Angelis, A. Gadea, Th. Kröll, N. Marginean, T. Martinez, F. Nowacki, Phys. Rev. C **69**, 034309 (2004).
24. A.F. Lisetskiy, R.V. Jolos, N. Pietralla, P. von Brentano, Phys. Rev. C **60**, 064310 (1999).
25. A.F. Lisetskiy, A. Gelberg, R.V. Jolos, N. Pietralla, P. von Brentano, Phys. Lett. B **512**, 290 (2001).
26. K.I. Ikeda, S. Fujii, J.I. Fujita, Phys. Lett. **3**, 271 (1963).
27. C. Gaarde *et al.*, Nucl. Phys. A **369**, 258 (1981).
28. E. Moya de Guerra, A.A. Raduta, L. Zamick, P. Sarriguren, Nucl. Phys. A **727**, 3 (2003).
29. S. Cohen, D. Kurath, Nucl. Phys. A **73**, 1 (1965).

30. W.A. Richter, M.G. Van Der Merwe,R.E. Julies, B.A. Brown, Nucl. Phys. A **523**, 325 (1991).
31. F. Ajzenberg-Selove, Nucl. Phys. A **523**, 1 (1991).
32. E.G. Adelberger, M.M. Hindi, C.D. Hoyle, H.E. Swanson, R.D. Von Lintig, W.C. Haxton, Phys. Rev. C **27**, 2833 (1983).
33. H.S. Wilson, R.W. Kavanagh, F.M. Mann, Phys. Rev. C **22**, 1696 (1980).
34. B. Jancovici, I. Talmi, Phys. Rev. **95**, 289 (1954).
35. I. Talmi, *Proceedings of the International Symposium on Electromagnetic properties Atomic Nuclei*, edited by H. Horie, H. Ohnuma (Tokyo Institute of Technology, 1984) p. 4.
36. A. Bohr, B.R. Mottelson, *Nuclear Structure*, Vol. **II** (Benjamin, New York, 1969).
37. R.J. Ascuitto, D.A. Bell, J.P. Davidson, Phys. Rev. **176**, 1323 (1968).
38. P. Wasilewski, F.B. Malik, Nucl. Phys. A **160**, 113 (1971).
39. Hubert de Guise, David J. Rowe, Nucl. Phys. A **636**, 47 (1998).
40. J.M. Irvine, *Nuclear Structure Theory* (Pergamon Press, Oxford, 1972).
41. A. Poves *et al.*, Nucl. Phys. A **694**, 157 (2001).
42. T.K. Onishi *et al.*, to be published in Phys. Rev. C (2005).
43. V.T. Koslowsky, E. Hagberg, J.C. Hardy, H. Schmeing, I.S. Towner, Nucl. Phys. A **624**, 293 (1997).
44. I. Reusen, A. Andreyev, J. Andrzejewski, N. Bijnens, S. Franchoo, M. Huyse, Yu. Kudryavtsev, K. Kruglov, W.F. Mueller, A. Piechaczek, R. Raabe, K. Rykaczewski, J. Szerypo, P. Van Duppen, L. Vermeeren, J. Wauters, A. Wöhr, Phys. Rev. C **59**, 2416 (1999).
45. H. Schatz, Phys. Rep. **294**, 167 (1998).